Can Math and Literature Mix in the Middle School?
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Is it possible to teach integrated units on mathematics and literature at the middle school? I don't mean this as a rhetorical question, but as a subject to explore. It's a question that motivated me in writing my book, *Lost in Lexicon: An Adventure in Words in Numbers*, due for release in October. Both as a child and as an adult, I had admired *The Phantom Tollbooth*, in which Milo visits Wisdom and tries to make peace between Digitopolis and Dictionopolis. Still, I had always felt that in *The Phantom Tollbooth*, the city of Digitopolis and the world of numbers received short shrift (there's a diminutive policeman named Shrift in the book), and I wanted to see if I could give numbers a fairer shake.

In preparing this session, I examined the mathematical ideas in three books, *Flatland* by Edwin Abbott, *The Phantom Tollbooth* by Norton Juster, and my own *Lost in Lexicon: An Adventure in Words and Numbers*. All three books address the difficulty and confusion that arise when we try to explain mathematics in words alone. In the case of *Flatland*, I think this quality makes the book an ideal text for pre-service teachers, to introduce them to the challenge of explanation.

Page numbers in this essay refer to the 1994 Harper Perennial paperback edition of *Flatland*, the current hardcover edition of *The Phantom Tollbooth*, and the upcoming 2010 Tumblehome Press paperback edition of *Lost in Lexicon*.

FLATLAND

Flatland was written in 1884 by Edwin Abbott, schoolmaster, theologian, Shakespearean scholar and mathematician, to introduce the concept of higher dimensions of space by immersing the reader in lower ones. In 1983 Isaac Asimov stated that "To this day, it is probably the best introduction one can find into the manner of perceiving dimensions." More, *Flatland* is a masterful satire of the hierarchies of Victorian society as well as of the limits our prejudices draw around our own understanding.

The narrator of *Flatland* is literally a Square, a satisfied Professional Man living in a two-dimensional world, on a plane. In his world, male social status is determined entirely by shape; the more sides a regular polygon has, the higher his status. The priestly class has so many sides as to be virtually indistinguishable from a circle. Only triangles may be isosceles instead of equilateral, and the sharper their most acute point, the lower their status. Women are lines, or at least such narrow parallelograms that they approximate lines, so sharp and difficult to see face on that they can damage other shapes. They must therefore use separate entrances from men, sway constantly so they can be seen, and utter a "peace cry" wherever they go.

One night the Square dreams of a visit to Lineland, a one-dimensional world where a series of short line segments move back and forth, identifying one another by sound. The Square tries valiantly to convince the king of Lineland that there is a higher dimension, but nothing can change that narrow-minded monarch's sense that the world he perceives is all that is possible.

Not long afterward, the Square is visited by a Sphere, an inhabitant of Spaceland, which is our familiar three-dimensional world. Because the Square can only perceive whatever shape intersects the plane of his flat world, as the Sphere descends through Flatland, the Square sees a point expanding into a circle of increasing diameter, and then decreasing into a point again before disappearing. Still, nothing the Sphere tells the Square about a third dimension or how he can see inside the inhabitants of Flatland proves convincing until the Sphere somehow anneals the Square to his surface and lifts him into the third dimension so he can see for himself.

Then, when the enlightened Square suggests that there may be other, even higher dimensions, the offended Sphere dumps him back in Flatland. There the Square tries to convince others of the gospel of three dimensions until he is imprisoned for heresy.

That's the outline of the simple plot of this slim, 120-page book. We can make literary hay of its satirical aspects, from its presentation of Lamarckian evolution to its discussion of plastic surgery, from its dismissal of women and workers as practically brainless to its description of how the upper classes subvert the rebellious tendencies of the downtrodden. But how might it be used in studying approaches to mathematics?

A theme that unifies both approaches to the book is the difficulty of rising out of one's own frame of reference—to see women as fully human, for example, or to believe that all creatures should have equal rights, or to comprehend the concept of a higher dimension. For Abbott the higher dimension may have had theological meaning; for mathematics it will suggest the multiple dimensions contemplated in relativity and string theory.

Here are some exercises that may be interesting to try with pre-service teachers or students of geometry.

Have one student read aloud the description of rainfall in Flatland in the middle of page 4, or of a typical house starting at the bottom of page 5, while a second student tries to draw the figures described at the blackboard or on an overhead. If the drawing student has difficulty, let other students offer advice on how to change the drawing.

Split into pairs, with one person taking the role of the Square and one the king of Lineland. Restating the debate found on pages 67-73, the Square should try to convince the king that a second dimension exists.

Switch sides and epeat the exercise for the Sphere trying to convert the Square to a three-dimensional view, as happens in the book on pages 79-88.

In a class discussion, work out by analogy the arguments a hypercube might use to convince the Sphere that a fourth dimension exists. For an illustrations of a rotating hypercube, see (with acknowledgements to Walter Seaman): http://www.math.uiowa.edu/~wseaman/DGImage5310022.htm#Hypercube%20images

As the Sphere descends through Flatland, he appears to the Square as a series of circles intersecting the plane. Try to describe how another three-dimensional figure might appear as it sinks through the plane. That is, how do a cube, a regular tetrahedron, or a cone, for example, intersect a plane? Consider and attempt to describe this series of appearances when the shape intersects the plane leading with a face, an edge, or a vertex, or in the case of the cone, with a variety of leading parts.

First use a verbal description to share ideas. Is there a difference when you think about moving the solid through the plane versus cutting a series of flat slices through the solid? Ask the students to imagine slicing through the solid with a cheese slicer.
Second, try to draw your intersecting shapes for one another.
Finally, use clay or another soft material and slice it to demonstrate the different shapes achieved.
For pre-service teachers, connect the idea of slices through the cone to the classic conic sections: circle, ellipse, parabola, and for a double cone, hyperbola. How does a triangular slice taken by dividing the cone lengthwise from point to base fit in with the conic sections?

> (A workshop member suggested this exercise of moving from words to drawing to manipulatives as a way to demonstrate to prospective teachers the need for multiple ways of representing ideas to make them accessible to the broadest number of students.)

THE PHANTOM TOLLBOOTH

Norton Juster, author of *The Phantom Tollbooth*, which was published in 1961, is an architect as well as a writer. The book tells the story of Milo, who is always bored, until the day he comes home to find a toy car and toy tollbooth. He drives the car into the windy roads beyond the border town of Expectations. During a series of misadventures, he picks up a faithful watchdog, Tock, and learns the sources of the country's troubles: the quarrelsome rulers of Dictionopolis and Digitopolis have banished the princesses Rhyme and Reason to the Castle in the Air.

Milo, Tock, and the unconvincing Humbug drive through the countryside encountering strange characters and having adventures that almost always involve puns and wordplay. They meet a man who is tallest or shortest, fattest or thinnest, depending on who you compare him to; Milo conducts an orchestra that paints the colors of the day; he steals a small word from a Soundkeeper who is hoarding all noise and making the Valley of Sound a Valley of Silence; and some careless thinking causes him to jump unexpectedly to the Isle of Conclusions. Finally he reaches Digitopolis, and on page 171, a mathematical section begins.

In Digitopolis, Milo passes a sign with some challenging distance conversions and meets the Dodecahedron, who has a different face for every mood. Milo and his companions visit the Number Mine where jewels are tossed aside in favor of numbers and broken numbers are used to make fractions. (This use for broken numbers is linguistically clever, but doesn't make a lot of mathematical sense to me. What does a shattered 4 become? A series of one-quarters? 8/2, equal to a single four?) They dine on Subtraction Soup, which makes them hungrier the more they ingest. But the most interesting issues, mathematically, arise on pages (188-193).

Consider the math problem on page 188. The Humbug, asked to solve the problem, immediately shouts out an answer—"17!" Milo thinks longer and comes up with zero. This might be a good problem to give students to work out at home before reading the relevant pages.

> The problem combines addition, subtraction, multiplication, and division, and the issue is that the answer is zero only if the problem is solved by proceeding methodically from left to right, the way we read, ignoring the order of operations we devote so much time to in middle school. Following the conventions of multiplying and dividing before adding and subtracting leads to an answer of -48 95/237. (On second thought, you might want to demonstrate this principle with some friendlier number combinations.)

This discrepancy in results can lead to a discussion of how the conventions for reading mathematics are different from those for reading fiction. You can't always read left to right or up to down. It's important to stop and try to figure a problem out, to study graphs and diagrams, to circle back for reinforcement or clarification, and to self-check constantly for understanding. A similar approach is required for math homework problems. It's difficult for students to recognize that the point of homework is not to finish as fast as possible but to self-check your understanding, challenge yourself, and practice until proficient.

On page 189 the question of "biggest" or "largest" or "longest" number arises, along with the concept of magnitude. This is another good place to comment on how common words may have different meanings in mathematics, and how the need for precision leads to specialized mathematical language.

A discussion of infinity grows naturally out of the discussion of big numbers. A teacher could ask one student to name the largest number she can think of, and then ask subsequent students to come up one at a time and write a larger number—or even a number just one larger, to approximate Milo's experience of climbing the infinite stairway.

On page 195, Milo meets .58 of a boy, the typical partial child of an average family with 2.58 children. This delightfully ridiculous fact can serve as a jumping-off point for the difference between mean, median, and mode, and which is the most appropriate to use when looking at discontinuous rather than continuous data. Students could be asked to use the partial boy's method to find the average student in a group or the whole class. What gender is the student? In what country was he/she born? When is the average child's birthday? What is the average child's favorite animal?

> When they have come up with a composite kid, ask the students to discuss other ways of finding the average (or typical) student and when different methods might be most appropriate.

A final bit of mathematical thinking appears in the trick Milo plays on the Mathemagician on page 200, in which Milo argues that since the Mathemagician and his brother Azaz are sworn to oppose whatever the other approves, they actually agree on something. Let students debate the logic of Milo's reasoning.

Some of the book's most enjoyable quotations come from this section, as when the Mathemagician tells Milo on page 198, "You'll find . . . that the only thing you can do easily is be wrong, and that's hardly worth the effort."

Milo responds by asking why it is that even the things that are correct often don't seem to be right.

LOST IN LEXICON

Lost in Lexicon relates the adventures of media-addled cousins Ivan and Daphne, who travel through their great aunt's barn cupola into Lexicon, a land of words and numbers. There, hopeful villagers promptly enlist them in a quest to find the country's missing children, who have wandered off, enchanted by lights in the sky. In search of the children, Daphne and Ivan travel between Word and Number Villages, solving problems and gathering clues. For example, they overcome a plague of punctuation, tame a wild thesaurus, befriend the Mistress of Metaphor, supply new meters to a poetry village, and try to mediate among feuding parts of speech.

In mathematics, Ivan and Daphne calculate area to fix a tile-making machine, try to teach a seamstress to measure, figure out the workings of an algebraic compass that uses slope to indicate direction, explore geometric transformations, and deal with the inhabitants of Irrationality.

While wothwhile mathematics lessons that could build around area and slope, tessellation and number systems, I want to focus on just a few of the cousins' encounters.

1. In pages 102-104, Ivan tries unsuccessfully to help the hapless seamstress, Miss Needle, understand measurement. (Note that Miss Needle, like the other women of the village of Merry Measure, speaks in iambic tetrameter.) These pages could be introduced by asking students to choose their own unit of measurement (one example would be their own foot) to measure a set of common distances. Students can then compare measurements and try to reconcile their answers. Can they find a reliable way of converting from one measure to another? Then ask students to read the section and discuss why Ivan has such trouble explaining measurement to Miss Needle, and what misconceptions she continues to embrace.

In pages 169 – 171, Ivan and Daphne try to gain entry to Irrationality by coming up with interesting questions and problems. Ask students if they can find anything interesting about the number 1729. Internet research may lead them to discover that 1729 is the subject of an anecdote about the mathematicians Hardy and Ramanujan, and that it is the smallest number that can be expressed as the sum of two cubes in two different ways. Students might like to come up with their own interesting numbers and an explanation of what's interesting about them.

In the same pages, Daphne struggles with the concept of infinity and the density of the rational numbers. Students may struggle with the idea that there are an infinite number of numbers between 0 and 1. A teacher might ask a student to come up with two such numbers, very close to one another, such as 1/22 and 1/23. Another student could be asked to come up with a number between these two, and so on.

In pages176-177, the young mathematician Bran and his friends try to reason out whether they should worry that the children of Irrationality, sent elsewhere for their early schooling, may have disappeared. Though they attempt to make logical arguments, their reasoning show many of the flaws common in attempts to apply propositional logic. Working out what's wrong with these arguments could be an extension activity for the whole class or for selected students.

In pages 187-190, the cousins consult the mathemystical Zeta about how they might reach the Land of Night. By this time they know that Lexicon is laid out on a Cartesian plane and that they wish to travel from quadrant I to quadrant II, though their way is barred by virtually impassable mountains. Zeta describes three possible methods of movement (or transformation) in the plane: translation, rotation, and reflection. Unfortunately, the mechanical means for the first two are unavailable, but she does help them reflect through a mirror and extend their journey on the far side by a fudge factor or scale factor that successfully transports them across the border.

> Students often begin their studies of transformation even earlier than middle school, and depending on their knowledge of geometry and algebra they can solve different kinds of problems involving translation, rotation, and reflection.
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> > a. They can figure out the direction and magnitude of translation in just the x or y direction, for example in moving from (3, 8) to (3, -2), or they can find the distance of translation in other directions (for example from (3, 8) to (-1, 5) using the Pythagorean theorem.

b. For reflections, they could estimate where to draw the plane of reflection between two points, or they could experiment how the sign of the x or y member of an ordered pair changes when a point is reflected over the x or y axis.

For rotation, students could consider rotation around the origin and explore how the sign of the x or y coordinate changes with rotations of 90 or 180 degrees in either direction. They could also estimate where the fulcrum of rotation could be placed to swing along a rigid arm from, say, (2, 2) to (-1, 1). Solving that problem precisely using algebra and geometry could also be a useful professional development exercise for teachers.

These are my speculations about the mathematical understandings and explorations that could arise from studying three different pieces of literature. I have not emphasized the literature half of the integration, but all three books share aspects of allegory, and the last two share a wealth of wordplay. There's plenty here to build integrated units around, and all three books were written in a way that rewards deeper delving.

Whether it's workable to take an integrated approach may be another question. Flatland may be well-suited to a pre-service math methods course, with the mathematics taking the primary role and the literary satire being more of an aside. The Phantom Tollbooth might be taught in a self-contained classroom in grades 5 or 6, or taught primarily as literature with some enrichment activities supplied by a math teacher. It seems to me that the best approach to teaching Lost in Lexicon would be in collaboration between math and English teachers, perhaps with the math teacher coming in once a week. An independent school near my home will be trying this approach with a group of sixth graders this year.

I'd be very interested in hearing if anyone tries teaching any of these books in preservice or middle school classrooms. I'd also love to hear about other books educators may think would be well-suited to some sort of integrated math and literature approach.